Journal of Power Sources 191 (2009) 16–21

Contents lists available at ScienceDirect

Journal of Power Sources

journal homepage: www.elsevier.com/locate/jpowsour

Valve Regulated Lead Acid battery float service life estimation using a Kalman filter

Webb L. Burgess*

Eaton Powerware Corp., 8380 Capital Boulevard, Raleigh, NC 27616, USA

ARTICLE INFO

Article history:
Received 26 September 2008
Received in revised form
25 December 2008
Accepted 31 December 2008
Available online 14 January 2009

Keywords:
VRLA battery
Lead acid battery
Float service life
Kalman filter

ABSTRACT

A Kalman filter is developed from a model which characterizes the float service life of a battery into two phases. Once the latter phase of the float service life, that time when the capacity begins to decrease rapidly, has been detected the Kalman filter is started. Outputs of the filter are a smoothed version of the battery capacity and the projected capacity at specified time intervals in the future. It is this project ahead step that is used to estimate the remaining float service life of the battery.

1. Introduction

The float service life of a battery can be divided into two distinct periods as is depicted in Fig. 1. The first period is that time during which the loss of capacity is small. This can be thought of as a threshold or guarantee time. The second is characterized by a much more rapid decrease of capacity over time and continues until there is no useable capacity remaining. The length of the first period is determined by several factors, one of which is the discharge rate used in the test or application. The lower the discharge rate the longer will be the time of this portion of float service life. The duration of the second period is governed more by the battery design and the particular mechanism controlling life.

In most float service applications, such as a UPS, an important feature of battery management is the ability to estimate the time remaining for the battery to reach end of life. One method, developed for this task, is to use the project ahead step in a Kalman filter loop to estimate the remaining life of the battery. With a suitable model of the capacity degradation process during the second period of float service life, it is possible, after each measurement of capacity, to estimate the capacity at a specified point in the future.

The use of a Kalman filter for state-of-charge (SOC) applications has been described in several published papers in recent years. Chief among these is the series by Plett [1–3] and Vasebi et al. [4]. The models assumed, to which the Kalman filter is applied, are for the most part based on known physical principles or properties of a particular battery chemistry. In some cases electrical circuit analogues of the electrochemical charge/discharge processes are developed for the battery chemistry of interest.

In contrast to the approaches summarized above the track taken in this work is based on observed behavior of the degradation of capacity of a VRLA battery in float service operation. A probability distribution is identified that matches this observed behavior. This distribution is recast as a system of linear differential equations from which the Kalman filter is obtained.

The sections following will describe the model to be used, the formulation of the Kalman filter from this model and results obtained applying this method to actual float service life data.

2. Capacity degradation model

In a float service life application, as a battery ages, two mechanisms govern the rate of degradation of capacity, grid corrosion and loss of electrolyte. It is important to develop an understanding of the process by which physical measurements, in this case the pairs (capacity, time in float service), can be incorporated into a probabilistic setting. The benefit is, it allows the methods of probability and statistics to be used to explore the data and perform analyses to determine whether a relationship exists between the measurements. Furthermore, it may be possible to identify the underlying distribution which might adequately describe the relationship.

Imagine now taking a sample, of some size, of a particular manufacturer's battery and placing them into the same float service application. At some interval, not necessarily periodic, the batter-

* Tel.: +1 919 431 6454.
E-mail address: webblburgess@eaton.com.

0378-7753/$ – see front matter © 2009 Eaton Powerware Corporation. Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.jpowsour.2008.12.123

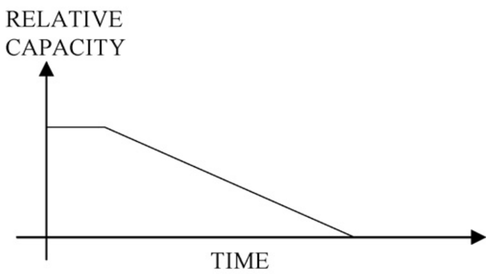

Fig. 1. Relative capacity vs. time of VRLA battery.

ies are discharged using a prescribed set of conditions (load and end voltage). The capacity (or discharge time) is measured and recorded together with the operating time of the battery. This is repeated until the capacity of each battery has diminished to a point where they are no longer useful. At the end of this exercise one will have a collection of battery capacities and operating times for each battery.

One possible use of the data is to make some estimate of the length of the float service life of this battery model. That is, how long will the battery operate before its capacity decreases to some specified value, generally given as a percentage of the rated or initial capacity. The first step in this process is to normalize the capacity (or discharge time) measurements. The method used here is to divide every capacity measurement, from each battery in the sample by a reference value, chosen generally from prior test data, to be slightly larger than the largest capacity in the sample. The result will be a collection of relative capacities, κ, where $0 < \kappa < 1$.

Consider now any of the particular points in time at which the sample of batteries is discharged. There will be a range of (relative) capacities measured that correspond to the time on test when the discharges were conducted. It is helpful to think of these different capacities as being due to different ages of the battery. A battery with a relative capacity of 0.83 has not aged as much as another in the sample with a relative capacity of 0.79. In other words the rates of aging are different for these two batteries since they have both been on float for the same amount of time. Hence one would expect the float service life of the first battery to be longer than that of the second. Continuing along this line of reasoning allows the age of the battery to be treated as a random variable.

Now it is possible to formulate the relative capacity and time on test in a probabilistic statement:

$$k_i = Pr\{L \leq L_i\} = 1 - Pr\{L > L_i\}. \quad (1)$$

The relative capacity equals the probability that the age of the battery, L, is less than or equal to the accumulated time on float at the ith discharge, L_i. The expression $\Pr\{L \leq L_i\}$ is the cumulative distribution function. The remaining work is to find a distribution whose properties match those of the data collected. In [5] the extreme value distribution was found to adequately represent the capacity degradation process. It is of the form:

$$F^{-1}(\kappa) = a_1(L - L_0) + a_0. \quad (2)$$

Here

(i) L_0 is the length of the first period of the float service life;
(ii) L is the age of the battery; $L - L_0 \geq 0$;
(iii) parameters a_1 and a_0 are estimated from the data;
(iv) $F^{-1}(\kappa)$ is the inverse distribution function, $F^{-1}(\kappa) = \ln[-\ln(1-\kappa)]$.

The model in (2) defines a random process. To design the Kalman filter a representation of the random process in terms of a system of linear differential equations must be developed first. This will be shown in the following section.

3. Kalman filter formulation

A Kalman filter is an algorithm for obtaining a minimum mean-square error point estimate of a random process. It is a method of least squares filtering that is obtained from a state space formulation. To start it is necessary to recast (2) as a system of linear differential equations. Note that (2) is just a linear equation in the variable L, with slope a_1 and y-intercept a_0. For this model let:

$$y(L) = a_1(L - L_0) + a_0. \quad (3)$$

Then carrying out the following steps let

$$x_1 = y(L)$$
$$x_2 = \dot{y}(L) = \dot{x}_1$$
$$\dot{x}_2 = \ddot{y}(L) = 0$$
$$y(L_0) = a_0$$
$$\dot{y}(0) = a_1$$

The resulting system of linear differential equations from these operations is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \quad (4)$$

The discrete version of the state transition matrix can be obtained from (4) and is of the form:

$$\phi(\Delta t) = \begin{bmatrix} u(\Delta t) & \Delta t \\ 0 & u(\Delta t) \end{bmatrix} \quad (5)$$

where $u(\Delta t)$ is the unit step function.

The Kalman filter equations or loop are listed in (6)–(10) and following these the initial conditions will be developed. Following initialization, the sequence of steps, (6)–(10) are executed in the order shown. After the last step the process is repeated using the quantities from the project ahead step as inputs to start the loop again.

Start filter

(1) Compute Kalman gain:

$$K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1} \quad (6)$$

(2) Update estimate with measurement z_k:

$$\hat{x}_k = \hat{x}_k^- + K_k[z_k - H_k \hat{x}_k^-] \quad (7)$$

(3) Compute error covariance for updated estimate:

$$P_k = [I - K_k H_k] P_k^- \quad (8)$$

(4) Project ahead:

$$\hat{x}_{k+1}^- = \phi_k \hat{x}_k \quad (9)$$

$$P_{k+1}^- = \phi_k P_k \phi_k^T \quad (10)$$

Some of the terms in (6)–(10) can be defined without too much explanation.

$z_k = \begin{bmatrix} y(L_k) \\ a_1 \end{bmatrix}$: measurement of normalized capacity, $y(L_k)$, and slope, a_1;

$x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix}$: state vector at t_k;

$H_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$: matrix defining the relationship between the measurement and the state vector.

R_k is the covariance matrix of the measurement error. That is, for each observation of the state, x_k, there are two components, the measurement, z_k, and an additive error term, v_k. The expression used here is $x_k = z_k + v_k$. The covariance matrix of the error is of the form:

$$E\left\{v_k v_i^T\right\} = \begin{Bmatrix} R_k, & i = k \\ 0, & i \neq k \end{Bmatrix}. \tag{11}$$

The resulting matrix, R_k, is

$$R_k = \begin{bmatrix} v_1^2 & 0 \\ 0 & v_2^2 \end{bmatrix} \tag{12}$$

The assumption is made above in (11) that the measurement error has zero mean (i.e. no fixed bias). The measurement error can now be thought of as just a noise term. It is defined, for the purpose of this work, as

$$v_1 = v_2 = 0.01. \tag{13}$$

It remains to obtain estimates of the error covariance matrix, P_k^-, of the state variable to use for calculating the Kalman gain (6), the first equation in the loop. The basic approach is to use whatever knowledge is available about the process (3) prior to entering the Kalman filter loop. First, the state estimation error is defined as

$$e_k^- = x_k - \hat{x}_k^- \tag{14}$$

Here, $\hat{x}_k^-$ is the estimate of the state just prior to time t_k. The error covariance can now be defined as

$$P_k^- = E\{e_k^- e_k^{-T}\} = E\{(x_k - \hat{x}_k^-)(x_k - x_k^-)^T\}. \tag{15}$$

Ignoring the noise term v_k for the time being and using just z_k (15) becomes

$$E\left\{\begin{bmatrix} Y_k^- - \hat{Y}_k^- \\ a_1^- - \hat{a}_1^- \end{bmatrix}\begin{bmatrix} Y_k^- - \hat{Y}_k^- & a_1^- - \hat{a}_1^- \end{bmatrix}\right\}$$

$$= E\left\{\begin{bmatrix} (Y_k^- - \hat{Y}_k^-)^2 & (Y_k^- - \hat{Y}_k^-)(a_1^- - \hat{a}_1^-) \\ (Y_k^- - \hat{Y}_k^-)(a_1^- - \hat{a}_1^-) & (a_1^- - \hat{a}_1^-)^2 \end{bmatrix}\right\} \tag{16}$$

The task now is to estimate values for each entry in (16) using prior knowledge of the process. Consider first the entry:

$$E\{(Y_k^- - \hat{Y}_k^-)^2\}. \tag{17}$$

Prior to starting the Kalman filter, three measurements of capacity will have been obtained. From these a linear, least squares fit to the data (i.e. (3)) can be calculated. To indicate this is a priori knowledge (3) can be rewritten as

$$\hat{Y}_k^- = \hat{a}_1^-(L_k - L_0) + \hat{a}_0^- \tag{18}$$

where the superscript minus signs now indicate values prior to starting the Kalman filter (i.e. prior to the next measurement) and $\hat{a}_1^-, \hat{a}_0^-$ are the estimated coefficients. Given the model above (18) each measurement of capacity satisfies:

$$Y_k^- = \hat{a}_1^-(L_k - L_0) + \hat{a}_0^- + e_k^-. \tag{19}$$

where e_k^- is the error between the data and the model. Fig. 2 shows the relationship between the model (i.e. fitted line) and the data.

The quantity e_k^- is often referred to as the residual. It is considered to have a mean of zero. Returning to (17), subtracting (18) from (19) yields

$$E\{(Y_k^- - \hat{Y}_k^-)^2\} = E\{e_k^{-2}\} = \frac{\sum_k e_k^{-2}}{n-1}. \tag{20}$$

This is the variance of the residuals which can be easily obtained from the a priori data.

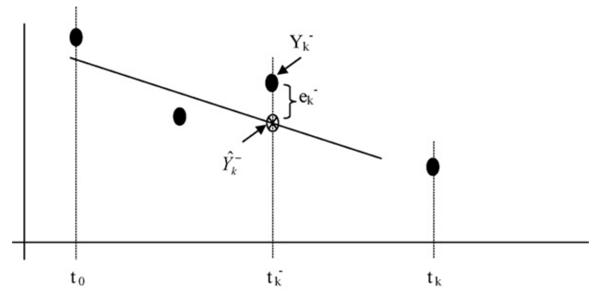

Fig. 2. Relationship between fitted line and data.

Next is the diagonal term:

$$E\{(a_1^- - \hat{a}_1^-)^2\}. \tag{21}$$

For the purpose stated above, obtaining starting values for the Kalman filter using prior knowledge of the process, think of $\hat{a}_1^-$ as the true or exact value of the slope. That is, $\mu_{a_1^-} = E\{a_1^-\} = \hat{a}_1^-$. Using this in (21) yields

$$E\{(a_1^- - \hat{a}_1^-)^2\} = E\{(a_1^- - \mu_{a_1^-})^2\}. \tag{22}$$

This is the variance of the estimate of the slope and is given by σ^2/S_{xx} ([6] Eq. 1.4.1; p. 24). This can also be calculated from the prior data.

The last quantity to be estimated is the off diagonal entries in (16). This is the covariance between the two variables. Using the following reasoning one might suspect the off diagonal terms are zero. The first term inside the brackets, $Y_k^- - \hat{Y}_k^-$, is the residual. It has a mean of zero. It has been shown that $cov(\bar{Y}, a_1) = 0$ ([6]; p. 28). For initializing the covariance matrix set the off diagonal entries to zero.

This concludes the setup and initialization of the Kalman filter algorithm. The implementation of the filter to estimate the remaining float service life will be described next.

4. Kalman filter implementation

The previous section provided the algorithm for the Kalman filter and the means for estimating the quantities needed to start the filter. In this section the implementation of the Kalman filter for use in predicting the end of the float service life of the battery will be described. The first task is to detect the start of the second period of the float service life as described in Section 1 and shown in Fig. 1. Until this period is detected the Kalman filter is not operating.

The state of the battery is obtained by performing a discharge at some convenient interval. For the batteries used in this work a discharge was performed approximately every 15 days at 50 °C. The discharge load was 17 A (2C) constant current to an end voltage of 1.83 V cell^{-1} (10.98 V). Discharge time for a new battery is approximately 1000–1100 s. At the end of each test discharge the (calendar) date and time are recorded together with the discharge time. The discharge time can be normalized by dividing by the reference time

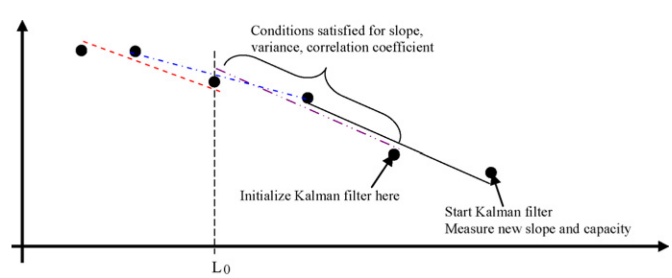

Fig. 3. Sequence of detection, initialization of Kalman filter and starting filter.

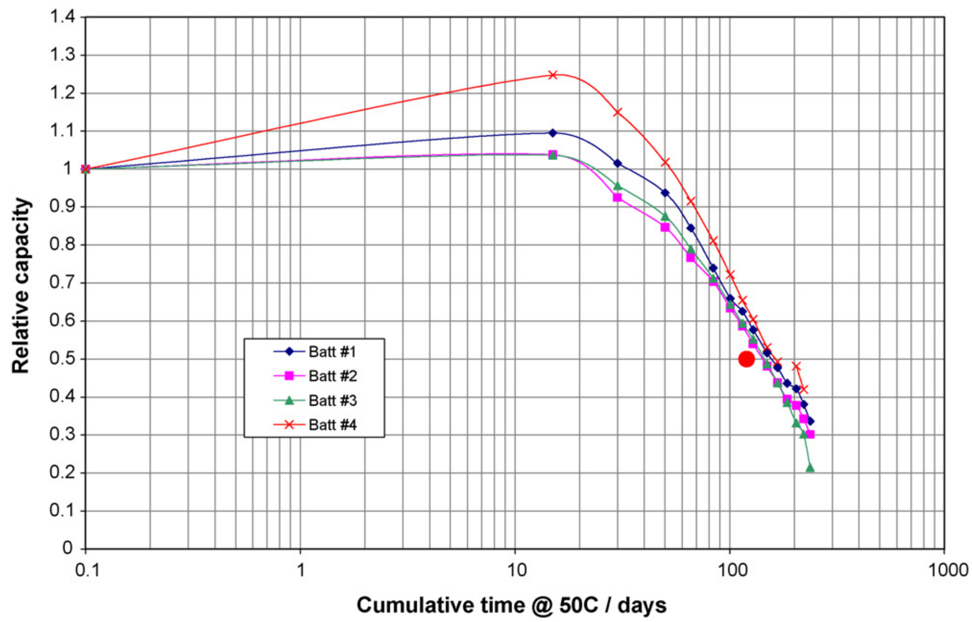

Fig. 4. Float service life test data.

chosen for the battery model and discharge performed. Here a time of 1200 s was used.

Once a series of three discharge tests have been accumulated the algorithm to detect the rapid change in slope of the capacity vs. calendar time (see Fig. 1) curve can be executed. This algorithm calculates a least squares fit to a straight line (2) using the variables $(L, \ln[-\ln(1-\kappa)])$ where L is the days in operation (referenced to the first discharge which is arbitrarily set to 0) and κ is the normalized discharge time. From the least squares fit calculation, the slope; a_1, y-intercept; a_0, and some measures of the quality of the estimate, variance; s^2, and correlation coefficient; r_{xy}, can be obtained. With these quantities, a comparison is made to threshold values of a_1, s^2 and r_{xy} to determine whether the battery has reached the stage where capacity starts to decrease rapidly. If all the above conditions are not met, then the process will be repeated when the next test discharge is performed. Upon completion, the first (earliest) test discharge will be removed and the latest one added to maintain three points for the least squares fit and slope detection algorithm. If all three conditions are satisfied the Kalman filter is started with the next test discharge.

The error covariance matrix (15), P_k^-, and the measurement error covariance (12), R_k, must be initialized first to obtain the Kalman gain (6). Here

$$P_k^- = \begin{bmatrix} s^2 & 0 \\ 0 & \dfrac{s^2}{S_{xx}} \end{bmatrix} \tag{23}$$

and

$$R_k = \begin{bmatrix} 0.01^2 & 0 \\ 0 & 0.01^2 \end{bmatrix}. \tag{24}$$

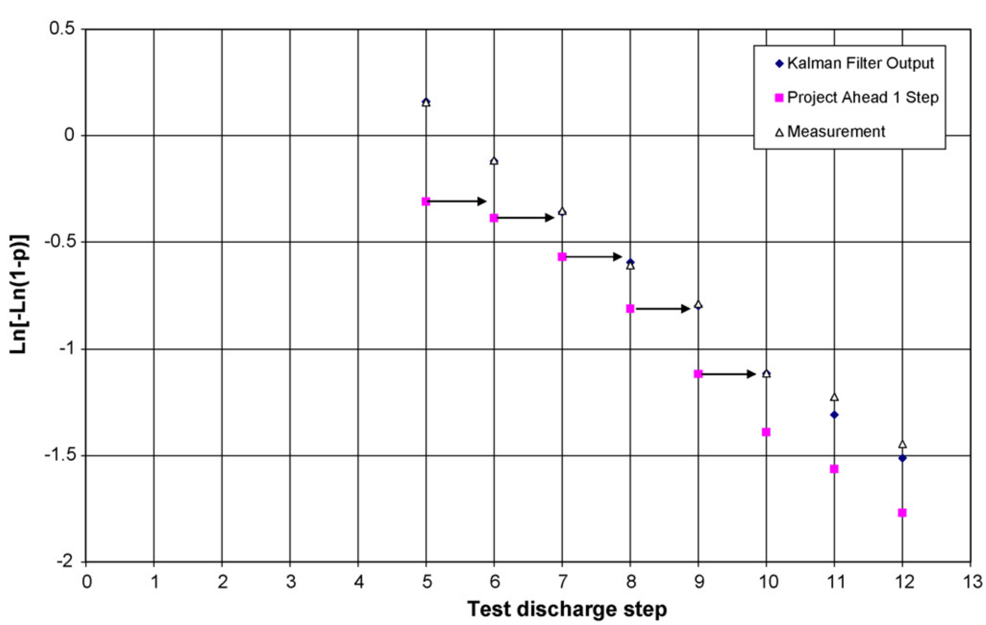

Fig. 5. Kalman filter performance of battery #1.

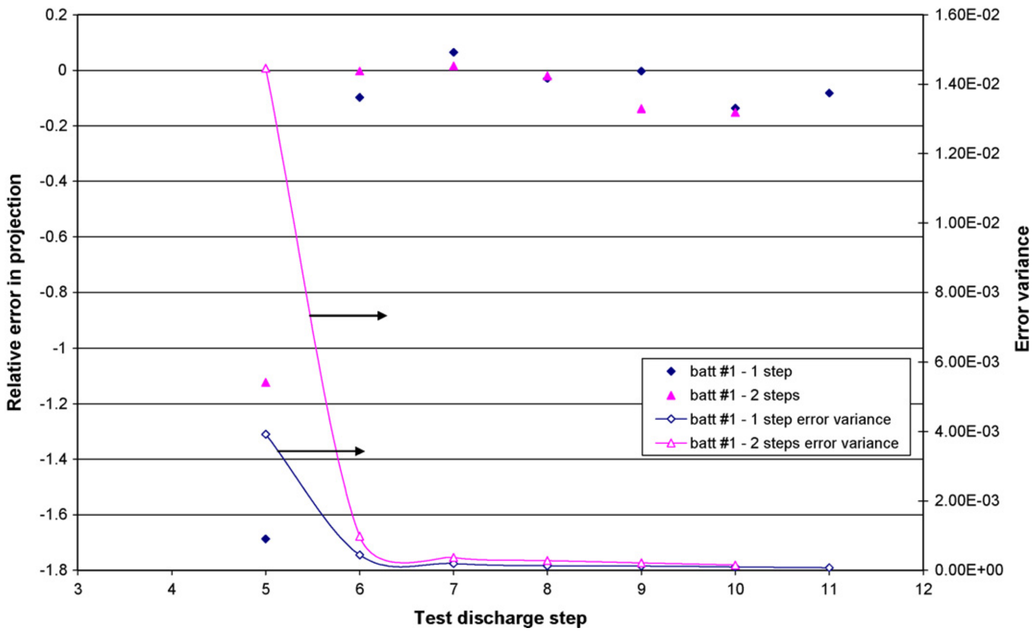

Fig. 6. Performance of project ahead step of battery #1.

Finally, the initial value of the state vector, $\hat{x}_k^-$, can be set using the results from the last test discharge when the three conditions on the slope were satisfied. A graphical depiction of initializing and starting the Kalman filter is contained in Fig. 3.

In this implementation of the Kalman filter, once the algorithm is started, the threshold time, L_0, is subtracted from the calendar time in all subsequent calculations. Time then is in hours since the degradation of capacity was detected.

The Kalman filter loop, as described in (6)–(10), can now be applied to the data from the most recent test discharge. The two main applications of this filter are to obtain a filtered estimate of the battery capacity after each test discharge is performed and to project ahead to determine the remaining time until the battery reaches end of life. The filtered capacity or estimate of capacity is from (7). The error covariance matrix (8) can be used to judge the quality of the estimate. The project ahead step in (9) and (10) can be used to estimate the remaining life of the battery.

Refer first to (5), where the elements of the state transition matrix are shown. The Δt entry is the time between the current or most recently performed test discharge and the next scheduled one. This step, (9) and (10), could be executed when the next test discharge is performed. To obtain an estimate of the life remaining chose a suitable time interval, Δt, of interest and compute an estimate of the state vector using (9) and the error covariance (10). In this work multiples of the time interval to the next test discharge were used. The time interval of the test discharges was approximately 90 days. Projections were made using intervals of 90 and 180 days.

5. Results

Float service life test data from four 12 V, 9 Ah VRLA batteries was used to test the performance of the filter for this application. The batteries were tested at 50 °C. Fig. 4 contains plots of relative capacity vs. days on test for the four batteries. The dot represents the float service life specification for this battery (120 days at 50 °C to 50% of initial capacity).

The first measure of performance of the filter is how well it tracks the battery capacity. Fig. 5 provides a visual indication of the performance of the filter. Here, the measurement data together with the filter output for battery #1 is shown. The horizontal axis, rather than being in units of time on float, is shown as discrete test discharges or steps. This will be used for all the graphs depicting filter performance. Note the agreement between the measured capacity and the filtered capacity for many of the steps is quite good. This shows in a qualitative way not only the performance of the filter but also the adequacy of the model chosen. The actual measure of performance used in this work is the relative error between the actual capacity and filtered capacity at each test discharge. Table 1 shows filter performance, in terms of this relative error, for each of the four batteries. The rms error is shown at the bottom of the table with and without the first data point. The reason for omitting the first data point is with some batteries, after detecting the slope change and starting the Kalman filter, the output has not stabilized. This can be seen from the results with battery #2 in Table 1. It is the worst performing battery of the four.

The second measure of performance of the filter is how well it can project the battery capacity at some point in the future. For this application this is the more important of the two performance measures. In particular, the interest here is how well does Step 4, (9) and (10), predict battery capacity at some specified time in the future. To obtain the relative error between the actual capacity and projected or estimated capacity the projections were made to the next data point and the succeeding one. In calendar time these projections were approximately 90 and 180 days in the future from each data

Table 1
Relative error between measurement and filter output.

n	Batt #1	n	Batt #2	n	Batt #3	n	Batt #4
5	−0.014			5	−0.088		
6	−0.042	6	6.051	6	−0.010	6	0.394
7	−0.018	7	0.542	7	0.008	7	0.045
8	0.022	8	0.108	8	0.059	8	0.029
9	−0.013	9	0.141	9	−0.006	9	−0.019
10	−0.001	10	0.015				
11	−0.070	11	−0.110				
12	−0.046	12	−0.130				
		13	−0.171				
rms error	0.035		2.15		0.048		0.20
1st data point removed	0.037		0.23		0.030		0.033

Table 2
Relative error in projections.

n	Batt #1	*n*	Batt #2	*n*	Batt #3	*n*	Batt #4
One-step ahead							
5	−1.687	6	0.562	5	−0.368	6	0.119
6	−0.0970	7	0.115	6	0.0410	7	0.0774
7	0.0654	8	0.155	7	0.171	8	−0.0443
8	−0.0294	9	0.0181	8	−0.0139		
9	−0.00275	10	−0.134				
10	−0.136	11	−0.165				
11	−0.0813	12	−0.222				
Two-step ahead							
5	−1.123	6	0.131	5	−0.258	6	0.145
6	−0.0026	7	0.160	6	0.199	7	0.0103
7	0.0159	8	0.0319	7	0.112		
8	−0.0197	9	−0.132				
9	−0.138	10	−0.190				
10	−0.151	11	−0.259				

point. The projections are shown graphically in Fig. 5. At each step the projection is made for the capacity at the next measurement. For comparison between the projection and measured capacity the solid rectangle in Fig. 5 is shifted right, to the next time step. The covariance at the projected step is calculated also. Both of these are used as inputs to the Kalman filter when it loops back to the beginning of the algorithm. The covariance from the projection is used in Step 1, (6) and the projected state is used in Step 2, (7). Note here the means by which the state vector is updated. With each new measurement the difference between the measured state and the projected state is taken, multiplied by the Kalman gain and added to the previous state. The Kalman gain, (6), is a function of the projected error covariance from the previous step.

The graph in Fig. 6 shows the relative error between the projection and the measured capacity for one and two steps ahead (90 and 180 days, respectively). The error variance is included also. Note the large error/large error variance for the first projection after the battery capacity degradation is detected. The errors tend to stabilize on succeeding measurements. Of significance is the relative error in the projections for both one and two steps ahead is under 0.20 (20%) through the end of life, typically near steps 8 and 9. It can be seen in Table 2 that this limit holds for all four batteries, after the first measurement, and both one and two-step projections.

6. Conclusions

The method developed here to estimate the remaining life of a VRLA battery takes advantage of the two distinct phases of the float service life. In the first phase, degradation of battery capacity is small, and hence no projections are made. Once the onset of the second phase of the float service life is detected, a Kalman filter is employed to track the degradation of capacity. The project ahead step in the filter algorithm is used to predict battery capacity at specific times in the future.

Using float service life test data from four batteries, it was demonstrated that the relative error in the predicted capacity was 20% or less for projection intervals of 90 and 180 days. The rms error in the filter output was less than 5% for three of the four batteries.

Initial conditions for the Kalman filter are obtained from measurements during the detection process. As a result the only knowledge required of the battery in use is a reference capacity used to calculate relative capacity.

Much of the published work on the application of a Kalman filter to estimate the state or condition of a battery is related to SOC and to a lesser degree state-of-health (SOH). To this writer's knowledge a Kalman filter has not been applied to the task of estimating the remaining float service life of a VRLA battery. One of the most recent methods on estimating the remaining life of a VRLA battery, which is not based on a Kalman filter, is provided by Pascoe in [7]. Here, battery life is estimated initially from calendar time on float plus operating temperature. Other factors such as float voltage and state-of-health indicators are incorporated, including occasional capacity checks, to calibrate the governing equations proposed to estimate the remaining life.

The method described in this paper provides similar results and obtains equivalent accuracy for the projections of end of life. Both methods require shallow discharges of the battery to obtain capacity estimates used as input to the algorithms. The main benefits of the approach described in this work are the Kalman filter can be initialized from measurements made during normal operation and an expected discharge time is the only information needed on the battery in use. It is expected then that the Kalman filter implementation described here will be more robust and require almost no characterization when applied to a wide range of battery models.

Acknowledgements

Special thanks are given to the technical management at Eaton Powerware for supporting this work. In particular it is with appreciation of this author to Lennart Jonsson for providing the inspiration to work on this project.

References

[1] G.L. Plett, Journal of Power Sources 134 (2004) 252–261.
[2] G.L. Plett, Journal of Power Sources 134 (2004) 262–276.
[3] G.L. Plett, Journal of Power Sources 134 (2004) 277–292.
[4] Amir Vasebi, Maral Partovibakhsh, S. Mohammad Taghi Bathaee, Journal of Power Sources 174 (2007) 30–40.
[5] W.L. Burgess, Journal of Power Sources 75 (1998) 28–35.
[6] N. Draper, H. Smith, Applied Regression Analysis, 2nd ed., Wiley, New York, 1981.
[7] P.E. Pascoe, A.H. Anbuky, IEEE Transactions on Energy Conversion 20 (December (4)) (2005) 887–895.